shall assume, for want of better information, that

$$T_i = V_i^3/l_i$$

$$T_d = (R\ )_d(V_d/l_d)^2\mu$$

Thus, since $T_i = T_d = T = (V_d/l_d)^2\mu$, the dissipation per unit mass is given by

$$T/\rho = \Delta = (V_d/l_d)^2\nu$$

We now assume that over the whole scattering volume $V_d/l_d = \Omega$ is a constant that is the frequency of the eddies characterized by the dissipation wave number K_d The magnitude of Ω is generally of the order of 10^3–10^4 sec^{-1} The Kolmogoroff spectrum is thus given by

$$E_E(K) = A\Omega^{4/3}\nu^{2/3}K^{-5/3} \equiv \phi_\epsilon(K)$$

and the radar cross section per unit volume is given by

$$\left\langle \frac{\sigma}{V} \right\rangle = \frac{A \sin^2\alpha\sigma_T N^2\Omega^{4/3}\nu^{2/3}K^{-5/3}}{1 + (\omega_c/\omega)^2}$$

where effects of collisions are approximated by the denominator (ω_c is the collision frequency) The coefficient A is a dimensional constant that cannot be predicted a priori, but whose magnitude can be determined empirically, as explained below Our previous discussion suggests that A is only weakly dependent on the detailed characteristics of the flow field

Comparison of Theoretical Predictions with Experiments

The formula for $\langle\sigma/V\rangle$ derived here was applied to the scattering from wakes of blunt bodies entering the atmosphere at velocities of the order of 7 km/sec The exponent for the frequency dependence over a decade was found to be between 1 67 and 1 71, as compared with the predicted value of $\frac{5}{3}$ The predicted altitude of the cross-section maximum at a particular frequency was within ±3 km of the observed altitude Both of these comparisons were made without specifying the coefficient A The coefficient A was determined by fitting the formula at the altitude of maximum return to that at the observed return Security restrictions do not permit going into greater detail here

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Radiative and Convective Heating During Hypervelocity Re-Entry

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Nomenclature

h_w = enthalpy of gas at wall
h_t = stagnation enthalpy
I = radiative emission per unit volume into 2π sr
q = heat-transfer rate
P_S = stagnation pressure
R_N = nose radius
V_∞ = flight velocity
ρ_∞ = ambient density

Because of the existing uncertainty with regard to the prediction of re-entry heating at superorbital velocities, the Ohio State University Aerodynamic Laboratory is engaged in a continuing study of both convective and radiative heat-transfer phenomena in high-enthalpy gases The facility being used in the experimental phase of this effort is the Ohio State University arc-driven shock tube facility, which is capable of generating hypervelocity flows possessing total enthalpies of more than 60,000 Btu/lbm [1]

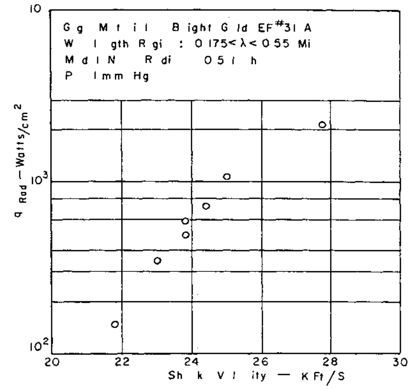

Fig 1 Experimental radiative heat-transfer data

Experimental measurements of stagnation-point radiative heat transfer are being carried out at conditions corresponding to those encountered by a vehicle during atmospheric re-entry at superorbital velocities The purpose of these radiative measurements is not to obtain a detailed picture of the spectral distribution of the radiative emission from high-temperature air, but rather to measure the integrated radiative flux Standard thin-film resistance thermometer techniques are being applied in this effort The material used in the present measurements for the sensing element of a thin-film radiative heat-transfer gage was Hanovia Bright Gold EF no 31-A ‡ This material is a liquid metal suspension; it may be applied to glass or ceramics by brushing, followed by firing in an oven to approximately 1250°F Specific details regarding the construction of resistance thermometer gages can be found in Ref 2 The spectral radiation characteristics of this gold material are such that only incident radiation at wavelengths below 0 55 μ will be sensed [3] In this region, approximately 75–80% of the incident radiation will be absorbed A complete discussion of this gage, its operating characteristics, and calibration procedures will be included in a future publication

For the present measurements, hemisphere-cylinder models, described in detail in Ref 4, were designed and constructed with the thin-film radiative heat-transfer gage recessed in a cylindrical well at the model stagnation point A sapphire window mounted on the model surface isolated the gage from the high-temperature shock-layer gas In this manner, the radiative heat transfer was separated out from the convective heat transfer It is important to note that the configuration of the recessed gage, the transmission characteristics of the sapphire window, and the absorptivity of the gage all must be taken into account in determining what percent of the stagnation-point surface radiative transfer is actually detected by the gage

To date, only a limited number of measurements have been made These have been at an initial driven tube pressure of 1 mm Hg using untreated room air and at shock velocities ranging from 21,750 to 27,750 fps The conditions in the shock layer thus correspond to flight conditions encountered at an altitude of approximately 110,000 ft and velocities ranging from 30,000 to 38,000 fps

The data obtained are presented in Fig 1 in the form of the stagnation-point radiative heat flux in the wavelength region of 1750 to 5500 Å as a function of the traveling shock velocity The radiative fluxes presented have been calcu-

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‡ Manufactured from Hanovia Liquid Gold Division, Engelhard Industries, 1 West Central Avenue, East Newark, N J

lated using a value of 0 82 for the average transmissivity of sapphire and a value of 0 80 for the average absorptivity of Bright Gold EF no 31-A in the wavelength region of observation

In analyzing the radiative heat-transfer measurements, which are purely of a preliminary nature, a comparison between theoretical estimates of the total radiative intensity of high-temperature equilibrium air[5–9] and the present measurements has been carried out This has been done by correcting the present measurements, which were taken over a limited wavelength region, to total intensities through the use of the wavelength distribution of the radiative intensity as predicted by Meyerott et al [5 8 9] In using the results of Meyerott, the radiative contribution due to N_2 $^+$ (1−) was corrected through the use of an f number of 0 04 [10]

In calculating the total radiative intensity from the present experimental results, it was assumed that the entire shock layer was in equilibrium, and the shock detachment distance was predicted using the experimental results of Ref 11 Based on presently available information on relaxation rates, the equilibrium assumption was justifiable

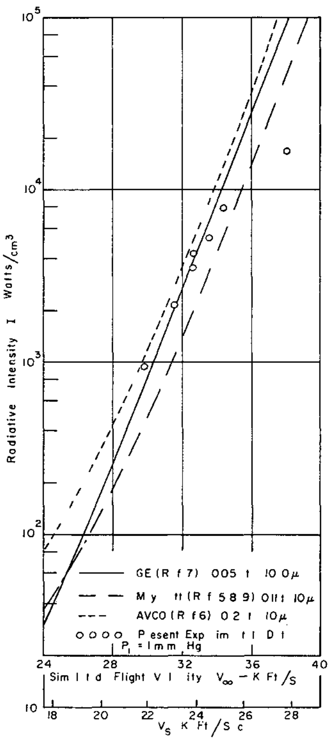

Fig 2 Radiative emission per unit volume of high-temperature air into 2π sr

The results of this analysis are shown in Fig 2 In general, the present experimental results are in good agreement with the theoretical estimate of Nardone et al [7] The corrected results of Meyerott are somewhat low, whereas the Avco results[6] predict too high a radiative intensity At simulated flight velocities above 34,000 fps, there appears to be a possible trend of the data falling away from the theory

An improved thin-film radiative heat-transfer gage is now in use in the Ohio State Laboratory This gage has an absorptivity of approximately 0 80 which is virtually independent of wavelength over the entire wavelength region of 0 17 to 6 μ Although results obtained with this gage are not fully available at the present, its use for engineering heat-transfer measurements appears promising The initial results do confirm the possible trend, shown in Fig 2, of much lower radiative transfer at extreme conditions than present theory predicts This would also be in agreement with the recent theoretical results of Biberman and Norman [12]

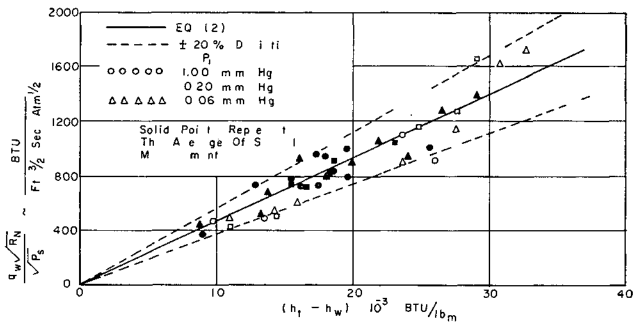

Fig 3 Correlation of high-enthalpy stagnation-point heat-transfer data in air

The limited amount of data certainly precludes the making of any definite conclusions concerning our present knowledge of shock-layer radiative emission during re-entry It does appear that, for dissociated gases with no appreciable radiation due to ionized atoms and electrons, present predictions are quite adequate However, for extreme conditions where the gas is partially ionized, our present theoretical estimates are extremely approximate For these conditions, there are even uncertainties concerning which species and processes produce the dominant emission [13] However, for engineering calculations, the theoretical results of Nardone et al [7] do appear conservative and should be used for the present

Extensive convective heat-transfer measurements have also been carried out for a three-dimensional stagnation point using a spherical model These measurements were made using a calorimeter-type heat-transfer gage[14] made of platinum foil 0 001 in thick The test conditions were such that the stagnation-point boundary-layer was believed to be in near equilibrium The results obtained are shown in Fig 3 in the form of a graph of $q_w(R_N^{1/2}/P_S^{1/2})$ as a function of $(h_t - h_w)$ The solid line is the equation

$$\dot{q}_w(R_N^{1/2}/P_s^{1/2}) = 47 \times 10^{-3}(h_{ts} - h_w) \qquad (1)$$

where q is in British thermal units per square foot-seconds, R_N in feet, P_S in atmospheres, and h in British thermal units per pound-mass It can be seen that the data scatter is within ±20% of the correlation curve, even out at high enthalpies where ionization is appreciable Thus, for engineering purposes, the foregoing equation should give satisfactory results, in addition to being practical It should be noted that the present results are in general agreement with those of Rose and Stankevics [15] It can also be seen that there are no serious ionization effects on stagnation-point aerodynamic heating at velocities up to 40,000 fps

If h_{ts} is replaced by $V_\infty^2/2$ and P_s by $\rho_\infty V_\infty^2$, then Eq (1) reduces to

$$\frac{\dot{q}_w R_N^{1/2}}{[1 - (h_w/h_{ts})]} = 20\ 5(\rho_\infty)^{1/2}\left(\frac{V_\infty}{1000}\right)^3 \qquad (2)$$

which is virtually identical to the approximate form of Lees [16] Here ρ_∞ is in slugs per cubic foot, V_∞ in feet per second, R_N in feet, and $\dot{q}_w$ in British thermal units per square foot-seconds

References

[1] Caldecott, R , Lee, J D , and Nerem, R M , "The development and application of an arc driven hypervelocity shock tube," 8th Midwestern Mechanics Conference, Case Institute of Technology, Cleveland, Ohio (April 1–3, 1963)

[2] Vidal, R J , "Transient surface temperature measurements, ' American Society of Mechanical Engineers Hydraulic Div Conference (May 21–23, 1962)

[3] "Ultraviolet, visible, and infrared transmission and reflection of hanovia metallic and luster films," Hanovia Liquid Gold Div , Engelhard Industries, Bull 22 (1962)

[4] Nerem, R M , "Measurements of aerodynamic and radi ative heating at superorbital velocities, ' Ohio State Univ , Aero dynamic Lab Rept 1598-1 (January 15, 1964)

[5] Meyerott, R E , "Radiation heat transfer to hypersonic vehicles," Third AGARD Colloquium on Combustion and Propulsion, Palermo, Sicily (March 17–21, 1958)

[6] Kivel B and Bailey, K , "Tables of radiation from high temperature air," Res Rept 21, Avco Research Lab (1957)

[7] Nardone, M C , Breene, R G , Zeldin, S S , and Riethof, T R , "Radiance of species in high temperature air, ' GE R63SD3, General Electric Space Sciences Lab (June 1963)

[8] Meyerott, R E , Sokoloff, J , and Nicholls, R W , "Absorption coefficients of air," GRD-TR-60-277, Geophysics Research Directorate, Air Force Research Div , Air Research and Development Command, Bedford, Mass (July 1960)

[9] Meyerott, R E , "Absorption coefficients of air from 6000°K to 18,000°K," U S Air Force Project Rand RM-1554, Rand Corp , Santa Monica, Calif (September 1955)

[10] Bennett, R G and Dalby, F W , "Experimental determination of the oscillator strength of the first negative bands of N_2^+," J Chem Phys **31**, 435 (1959)

[11] Graber, B C , "An experimental study of real gas effects on shock detachment distances and shock shapes for a group of spherically blunted bodies," M S Thesis, Ohio State Univ (November 1963)

[12] Biberman, L M and Norman, G E , "On the calculation of photoionization absorption," Opt i Spektroskopiya **8**, 433–438 (1960)

[13] Allen, R A and Textoris, A , "New measurements and a new interpretation for high temperature air radiation," AIAA Preprint 64-72 (January 1964)

[14] Rose, P H , "Development of the calorimeter heat transfer gage for use in shock tubes," Avco-Everett Research Lab , Res Rept 17 (February 1958)

[15] Rose, P H and Stankevics, J O , "Stagnation point heat transfer measurements in partially ionized air," IAS Paper 63-61 (January 1963)

[16] Lees, L , "Recovery dynamics—heat transfer at hypersonic speed in a planetary atmosphere," *Space Technology*, edited by H Siefert (John Wiley and Sons, Inc , New York, 1959), Chap 12, p 12 07

Dynamics of the Solar Wind

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Introduction

IT is now generally accepted that the solar atmosphere has an enormous extent and that it is responsible for many interplanetary phenomena Chapman,[1] in 1957, first studied the temperature distribution of such an atmosphere which he assumed to be static but thermally conducting Parker[2] criticized this static model and suggested instead a model in which energy is transferred outward from the coronal base by hydrodynamic streaming In this model, the fluid is accelerated outward by pressure gradients so that it arrives at the earth with a radial streaming velocity around 300–500 km/sec Thus, the "solar wind" observed near the earth is explained as a manifestation of the expanding corona Chamberlain[3] studied the fluid equations for a thermally conducting atmosphere with convection and heated only in a thin shell at the base Recently, Noble and Scarf[4] have solved these equations with boundary conditions appropriate to the solar wind; their results leave little doubt that Parker's model of a steady-state solar atmosphere with supersonic streaming at large radial distances is substantially correct For a general discussion of this problem together with a reasonably complete list of references, see Scarf's discussion [5]

In this note the earlier treatment is extended by inclusion of appropriate viscous dissipation terms in the hydrodynamic equations and by development of additional solutions for the lower corona At some intermediate distance from the solar surface, as the fluid becomes less dense, it is shown that the usual form of the transport coefficient equations becomes inapplicable, and, ultimately, the continuum equations themselves break down, and a form of free molecular flow sets in This transition to free or field-dominated flow is discussed and treated approximately

Lower Corona

Parker[6] considered the transport properties of the solar atmosphere and demonstrated that the lower corona behaves as a fluid; thus, the usual continuum equations apply These equations are discussed in detail along with the equations for the transport coefficients in an article by Burgers [7] We apply them here to a steady-state fluid in a gravitational field and assume spherical symmetry With these simplifications, the continuity equation is

$$(d/dr)(\rho ur^2) = 0 \tag{1}$$

where ρ is the mass density of the fluid and $u(r)$ is the radial streaming velocity We assume the gas to be neutral with 10% ionized helium and 90% ionized hydrogen, although there is some uncertainty at present regarding this concentration [4] For this case, the effective mass m of the gas particles is 0 62 times the proton mass and $N = 0\ 525\ N$ (total) The momentum and energy equations are, respectively,

$$\rho u \frac{du}{dr} + \frac{dp}{dr} + \rho \frac{GM_s}{r^2} = \frac{4}{3}\frac{d\eta}{dr} r \frac{d}{dr}\left(\frac{u}{r}\right) + \frac{4}{3} \eta \frac{d}{dr}\frac{1}{r^2}\frac{d}{dr}(r^2u) \tag{2}$$

and

$$\frac{1}{2} mu^2 - \frac{GM_sm}{r} + \frac{5}{2} kT - \frac{m\kappa}{(\rho ur^2)} r^2 \frac{dT}{dr} - \frac{4}{3}\frac{m\eta}{(\rho ur^2)} r^2 \left(u\frac{du}{dr} - \frac{u^2}{r}\right) = \text{const} \tag{3}$$

where we have assumed an equation of state

$$p = \rho/mkT \tag{4}$$

Here the usual fluid dynamic approximation for the transport coefficients has been adopted, i e , the heat flow vector **Q** is given by

$$Q = -\kappa(T)\nabla T \tag{5}$$

and the viscous stress tensor τ^{ij} is proportinal to the strain,

$$\tau^{ij} = \eta\epsilon^{ij} \tag{6}$$

Burgers'[7] derivation of the general transport equations from the microscopic viewpoint shows that Eqs (5) and (6) are approximations valid only where the temperature and velocity

Table 1 Predicted coronal parameters

λ	r/R_s	u, km/sec	N_e/cm^3	T °K
6 04	1 58	9 5	$6\ 21 \times 10^6$	$1\ 5 \times 10^6$
4 81	1 99	18	$2\ 10 \times 10^6$	$1\ 47 \times 10^6$
2 05	4 67	75	$9\ 20 \times 10^4$	$1\ 02 \times 10^6$
1 32	7 25	110	$2\ 56 \times 10^4$	$8\ 70 \times 10^5$
1 00	9 56	132	$1\ 22 \times 10^4$	$8\ 10 \times 10^5$
0 696	13 7	161	$5\ 0 \times 10^3$	$7\ 35 \times 10^5$

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